	<p>NUMERICAL-EXPERIMENTAL EVALUATION OF A HYPERELASTIC POLYURETHANE ADHESIVE</p>	<p>MECHANICS</p>
<p>COLLABORATION</p>	<p>Adi Corrales-Magallanes, Luis del Llano-Vizcaya, Celso Hernández Cruz-González, Vicente Bringas-Rico, Aldo Augusto López-Martínez, Eusebio Jiménez-López</p>	

NUMERICAL-EXPERIMENTAL EVALUATION OF A HYPERELASTIC POLYURETHANE ADHESIVE

<p>¹Adi Corrales-Magallanes, ¹Luis del Llano-Vizcaya, ¹Celso-Eduardo Cruz-González, ¹Vicente Bringas-Rico, ¹Aldo-Augusto López-Martínez, Eusebio Jiménez-López²</p>
<p>¹Centro de Ingeniería y Diseño Industrial (CIDESI), Departamento de Sistemas Automatizados, Av. Playa, Av. Pie de la Cuesta 702, Desarrollo San Pablo, 76125, Santiago de Querétaro, Querétaro. Tfno: +52 1 4422119800</p>
<p>²Universidad Tecnológica del Sur de Sonora-ULSA Noroeste-IIMM, Cuerpo Académico CIAAM, Calle Dr. Norman E. Borlaug km 14, 85190, Cd. Obregón, Sonora, México. Tfno: +52 1 644104312</p>
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1. INTRODUCTION

Adhesive bonding, is a technology in which one of its advantages is that it can join dissimilar materials. However, for the safe operation of bonded components, it is necessary to know their mechanical behavior in order to establish suitable operating parameters [1]. Several models are used to predict mechanical properties such as critical distance method [2]. However, the analysis will depend on the material itself. Hyper-elastic materials have a high deformability and nonlinear behavior between the load and its strain. Based on a phenomenological approach, these materials are conceived as a continuum, and it is taken into account that the tension energy density describes their hyper-elastic behavior [3].

Different constitutive models have been proposed to predict these characteristics, some of which require complex research to determine the parameters of the materials. One of the most used models to mechanical analyze elastomers is that of Mooney-Rivlin [4]. Mooney proposed the first and one of the most significant phenomenological theories of hyperelasticity in conjunction with Rivlin. The model expresses the deformation energy as a function of the elastic constant and the invariants of first and second stretching [5]. However, the model is only one of the infinite deformation energy functions for isotropic and incompressible solids that show a linear relationship between the shear stress and shear amount, and between the torque and the amount of torque, when subjected to large simple deformations by shear or torsion [6]. The Neo Hooke model is a particular case of the Mooney-Rivlin model [5] that is; it is use whenever the constant associated with the second invariant is equal to zero. The Ogden model proposes that it is possible that the stress energy density function can be written directly in terms of the main stretches [5]. Further information can be review in the following references [7,8,9].

In this article, the mechanical properties of polyurethane adhesive from are studied. The convergence of the Mooney-Rivlin, Neo Hooke, Ogden and Yeoh models was evaluated using the finite element method in Ansys V17.1 software, subsequently uniaxial stress tests were performed on specimens manufactured from the material in order to obtain the real mechanical properties. With the data obtained, a curve adjustment method was applied in order to generate a function and feed it to new numerical tests. This article presents a methodology to evaluate and select the model that best fits the experimental data for the selected adhesive, which can be extrapolate to the characterization of hyper-elastic models based on a uniaxial experimental test.

2. MATERIALS AND METHODS.

2.1 MATERIALS

The selected adhesive for these tests is a polyurethane adhesive with the following typical properties:


- | | |
|------------------------------------|---|
| • Base: polyurethane | • Tensile strength after 30 days: 475 psi |
| • Viscosity: Paste extrusion grade | • Ultimate elongation after 30 days: 350% |
| • Weight / Gallon: 9.8 pounds | • Shore a hardness after 30 days: 50 |
| • Specific Gravity: 1.17 | • Tack-free at 77°F, 50% RH: 40 minutes |
| • Temp. Range: -40°F to 200°F | • Lap shear: 400+ psi |
| • Shelf life: 12 months (unopened) | |

The adherent's basic properties are typical and extensively reported.

Adherent A

- Material: Borosilicate glass
- Modulus of elasticity: 82 GPa
- Shear modulus: 34 GPa
- Poisson's ratio: 0.206

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- Density: 2.51 g/cc

Adherent B

- Material: 1018 steel
- Modulus of elasticity: 205 GPa
- Shear modulus: 80 GPa
- Poisson's ratio: 0.290
- Density: 7.87 g/cc

The reference premises are:

1. The stress and strain states are a function of the strain energy, specifically, of the principal stretch rates or of the invariants of the Cauchy-Green tensor.
2. The adhesive behaves as a hyperelastic material
3. The interfaces between the bodies are considered as a continuum.

2.2 MECHANICAL TESTING

The specimens were built by molding at room temperature and vibrating the mold to remove air. After demolding them, they were visually inspected for external discontinuities, and the selected ones were left still at room temperature for 30 days.

The experimental test was performed at uniaxial tension with measurement in the central part of the specimen. The test was performed in a universal testing machine Instron 3365 at room temperature (20°C) with 50% HR. The test is load controlled. The strain rate is quasi-static, so that thermal effects (heat development) caused by a high strain rate and viscoelastic effects can be neglected. It was assumed that the material is incompressible and that the strain is independent of time. The force applied for each time step is recorded, as well as the displacement of the extensometer until the material fails. Fifteen specimens were tested, discarding some that presented irregularities in their curing.

The data obtained from the laboratory tests was compared with each other to determine the repeatability of the specimens. The data of 5 specimens that shown a good correlation were selected, (see Figure 1).

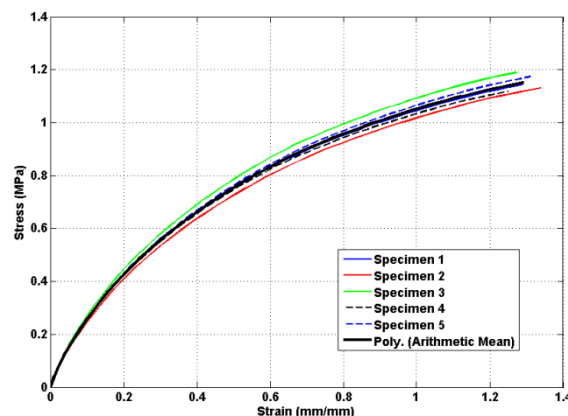



Figure 1. Stress-Strain plot: experimental data and Curve fitting (y is the stress value)

In order to obtain a single representative curve for the material, average of test specimens was calculated. From this averaged curve, an interpolation polynomial was obtained to smoothly depict the stress-strain curve of the material.

An approximation polynomial obtained from the arithmetic mean of the experimental data, was use to generate a table of values to fed into the software of hyper-elastic model curves. The above, with continuous data and without abrupt changes in stress values against strain.

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2.3 VARIATIONAL MODELS FOR THE STUDY OF HYPERELASTIC MATERIALS

2.3.1 CONCEPTS OF CONTINUUM MECHANICS

Below are some basic concepts of the continuum mechanics, as well as some fundamentals of hyperelasticity theory. The continuum mechanics is based on the definition of displacements and deformations in space. This can be defined by a vector representation as follows:

$$x_i = X_i + u_i \quad (1)$$

Vector x_i represents the actual position vector while X_i represents a reference position vector. Both are related through a displacement vector u_i . From the differential form of the previous equation, it can be reduced to the following expression:

$$dx_i = \frac{\partial x_i}{\partial X_i} dX_i = F_{ij} dX_i \quad (2)$$

Where F_{ij} represents the deformation gradient tensor.

The right Cauchy-Green tensor can be derived from the deformation gradient tensor as:

$$C_{ij} = F_{mi} F_{mj} \quad (3)$$

From (3), the following invariants can be defined, which are function of the stretch rates (λ):

$$\begin{aligned} I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \text{tr}(\mathbf{C}) \\ I_2 &= \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 = \frac{1}{2} (I_1^2 - \text{tr}(\mathbf{C}^2)) \\ I_3 &= \lambda_1^2 \lambda_2^2 \lambda_3^2 = J^2 = \det(\mathbf{C}) \end{aligned} \quad (4)$$

Strain can be defined from two perspectives: one is the nominal or engineering strain that can be defined as the change of a reference length $\Delta L = L - L_0$ and can be obtained from the principal stretch rates:

$$\varepsilon_i^i = \frac{\Delta L}{L_0} = \lambda_i - 1 \quad (5)$$

The logarithmic strain or Hencky strain is the second way to define it and can also be obtained through the principal stretch rates as follows:

$$\varepsilon_i^l = \ln(\lambda_i) \quad (6)$$

The stiffness is determined from the ratio between the load applied to the specimen and the resulting displacement for the load:

$$K_i = \frac{f_i}{|u_i|} \quad (7)$$

2.3.2 HYPERELASTIC MATERIAL MATHEMATICAL MODELS

There are several mathematical models to describe the behavior of hyperelastic materials, of which not all describe the behavior of a material in the same way. The selection of the most appropriate model is essential to achieve conclusive results in the numerical analysis. The models generally describe the hyperelastic properties of a material as a function of the strain energy density. This is:

$$W(\lambda_1, \lambda_2, \lambda_3) = W(I_1, I_2, I_3) \quad (8)$$

Some traditional hyperelastic models are summarized in the following lines. [7,8,9]:

1) Neo Hooke:


$$W = \frac{\mu}{2} (I_1 - 3) + \frac{1}{d} (J - 1)^2 \quad (9)$$

2) Blatz-Ko:

$$W = \frac{\mu}{2} \left(\frac{I_2}{I_3} + 2\sqrt{I_3} - 5 \right) \quad (10)$$

3) Yeoh:

$$W = \sum_{i=1}^N C_{io} (I_1 - 3)^i + \sum_{k=1}^N \frac{1}{d_k} (J - 1)^{2k} \quad (11)$$

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4) Mooney-Rivlin:

$$W = \sum_{mn=0}^{\infty} C_{mn} (I_1 - 3)^m (I_2 - 3)^n + \frac{1}{d} (J - 1)^2 \quad (12)$$

5) Ogden:

$$W = \sum_{i=1}^N \frac{\mu_i}{\alpha_i} (\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3) + \sum_{k=1}^N \frac{1}{d_k} (J - 1)^{2k} \quad (13)$$

To carry out the research in this work, the models of Mooney-Rivlin, Ogden and Yeoh, with different order and parameters were took into account for their analysis.

2.4 FINITE ELEMENT SIMULATIONS

One part with three continuous bodies is modeled and meshed with a structured mesh, where all the elements are quadrilateral.

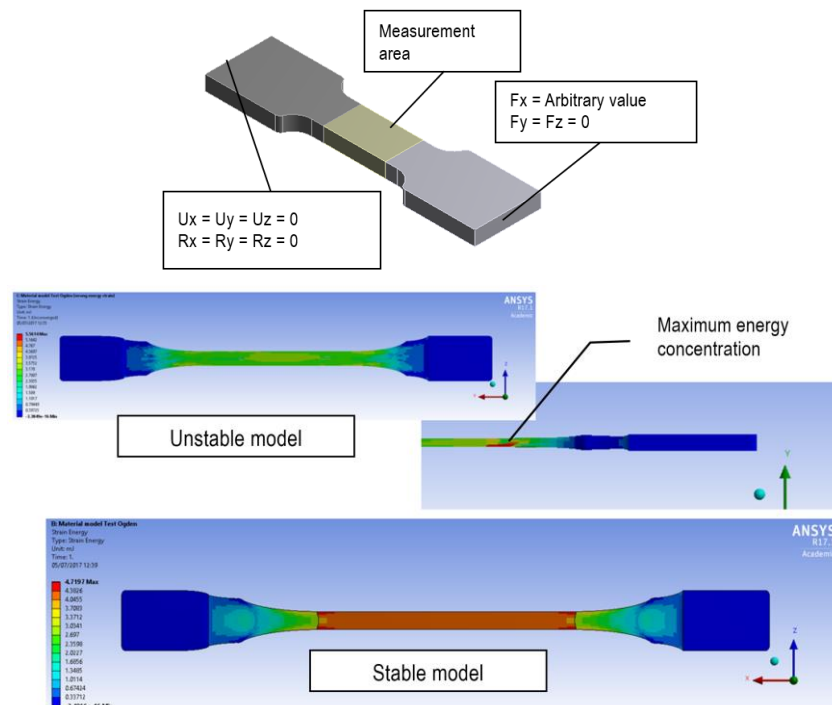



Figure 2. Boundary conditions and discrete model(top). Sample of deformation energy (mJ) for the Ogden Preliminary model (bottom)

Boundary conditions were defined with a fixed end while an arbitrary load is applied at the other end. A static structural analysis was performed.

This model was used to validate the functionality of the applied conditions, as well as the definition of the discrete model. The objective is to obtain a stable model, without artificial concentrations of energy, as shown in previous figure. An stable model should not present non-distributed concentrations of energy and this allows the validation of the discrete model and the analysis parameters. The unstable model shows unusual large deformations in the elements whose deformation energy is maximum.

From this experimental data, the software calculates through iterative techniques the parameters and characteristics of the material necessary for each model, and presents three continuous curves of the behavior of the material. In this case, a greater coincidence between the dotted curve (experimental data) and the blue continuous line (uniaxial behavior) represents a better description of the hyperelastic behavior by the model.

The following figure presents two models from which a qualitative analysis is performed One shows little correspondence between the experimental data and the adjustment of the uniaxial curve based on those models. These kinds of models they were discarded as valid models to describe the behavior of the polyurethane.

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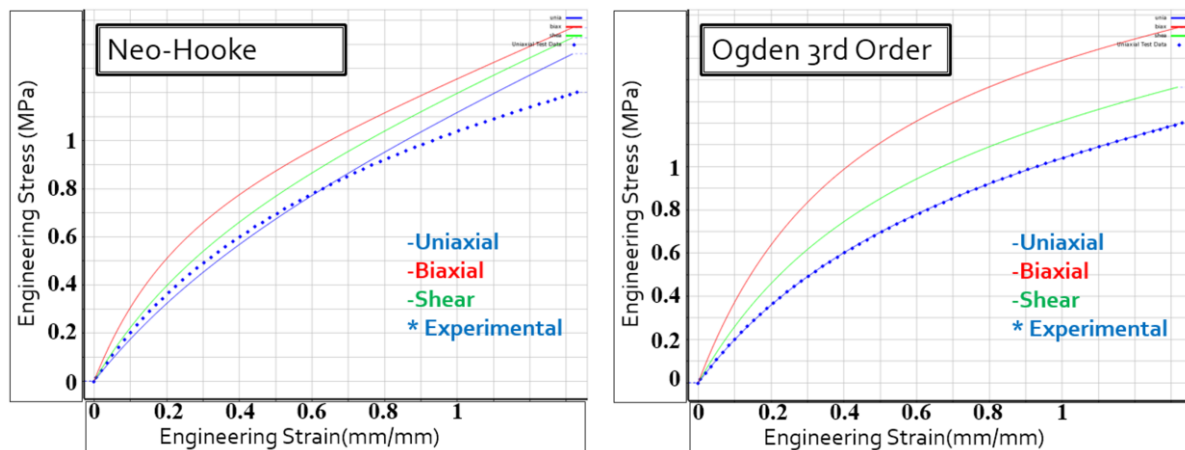


Figure 3. Curve fitting for non-fitting and fitting models

It is important to take into account the mathematical order of the models when fitting the curves, since the same model with lower order may not present an adequate fit, but may present better correlation with respect to the experimental results. Of the studied models, the 1st order Yeoh model does not have an adequate correlation, so it is presumed that it will not accurately represent the mechanical behavior of the material. This model was maintained as a control to evaluate a higher order Yeoh model. The Mooney-Rivlin model does not contemplate a mathematical order, but is classified by the number of parameters as, two, three, five, and up to nine. The model evaluated is that of two parameters.

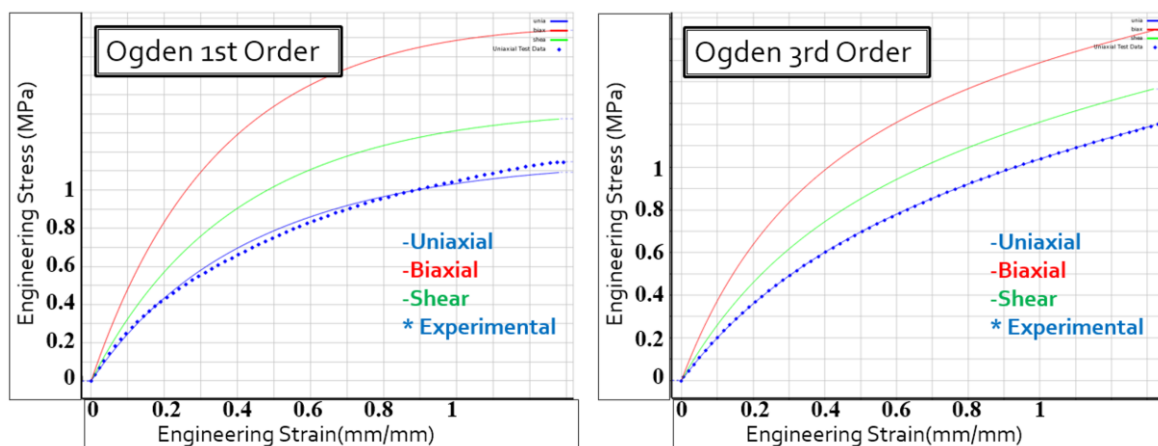


Figure 4. Ogden models of different order

The previous Figure also describe the fit for 3rd order Yeoh and 3rd order Ogden models. These models present a satisfactory adjustment with respect to the experimental data, being considered as candidates to describe the hyperelastic behavior of the material.

From these plots is possible to address a qualitative review to define the candidates for posterior analyses. Neo-Hooke and Blatz-Ko were considered deficient and discarded for further analyses. Yeoh 1st order is considered as deficient but kept as a control model to compare the higher order Yeoh models. Yeoh 3rd order, Ogden 1st order, Ogden 3rd order and Mooney Rivlin two parameters are considered acceptable and considered as candidates for the numerical model.


For the acceptable models, the parameters used in the model are as following:

Yeoh 3rd order:

- $C10 = 0.40622 \text{ MPa}$, $C20 = -0.039598 \text{ MPa}$, $C30 = 0.0041028 \text{ MPa}$
- Incompressibility parameters $1,2,3 = 0$

Ogden 1st order:

- $\mu_1 = 2.2498 \text{ MPa}$ $\alpha_1 = 0.76692$
- Incompressibility parameter $1 = 0$

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Ogden 3rd order:

- $\mu_1 = 0.010726 \text{ MPa}$ $\alpha_1 = 4.4317$, $\mu_2 = 9.6741 \text{ MPa}$ $\alpha_2 = 0.091084$, $\mu_3 = 9.7034 \text{ MPa}$ $\alpha_3 = 0.090917$
- Incompressibility parameters 1,2,3 = 0

Mooney-Rivlin 2 parameters:

- $C10 = 0.11846 \text{ MPa}$, $C01 = 0.36142 \text{ MPa}$
- Incompressibility parameter 1 = 0

3. RESULTS AND DISCUSSION

The numerical results were compared against the experimental data to obtain the model that best suits the stress test. In following figure, it can be seen which models were in good agreement against the data from the experimental tests.

It is also possible to observe that the higher order models and the 2 parameter Mooney model show an increase in the rigidity of the material when approaching the maximum load, since for the same stress value, a smaller deformation is observed, while the 1st order Ogden model maintains rigidity until it reaches an appropriate correspondence between stress and strain. To analyze the above in a clearer way, a graph of point residues of the stresses for the same strain is generated, comparing the experimental effort minus the numerical stress. Each step of loading the numerical models were plotted. A value farther from the abscissa represents a greater deviation from the numerical model with respect to the experimental one, whereas a negative value represents an increase in the rigidity of the material.

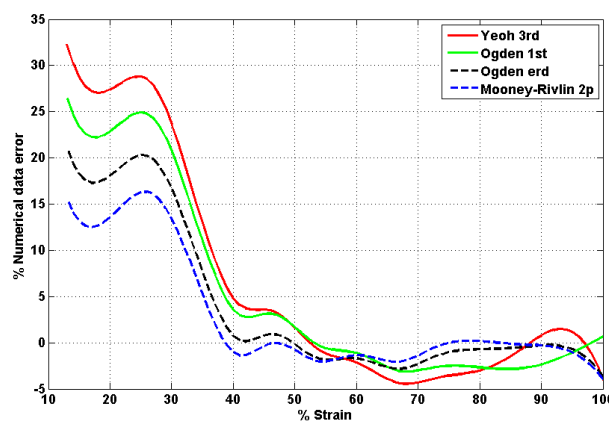



Figure 5. Comparison of hyperelastic models and experimental results (top) and punctual stress residues between hyperelastic models (bottom left) and error of the numerical results against experimental data (bottom right).

From the previous figure it can be seen that the model with larger deviations is the Yeoh model, while the two-parameter Mooney-Rivlin model and the 3rd order Ogden model have areas with residuals closer to the experimental data, suggesting this that both better represent the characteristics of the material in a uniaxial stress test. However, as the load approaches the maximum load, the material shows an increase in stiffness, while the 1st order Ogden model is more in line with the experimental data.

Previous figure also shows the percentage of deformation error obtained numerically against the experimentally obtained values, for the same deformation value. A negative percentage represents an increase in the rigidity of the material.

It is important to highlight that the initial deviations are due to the variations of the instrumentation in hyperelastic materials. The system must stabilize at the start of the test to begin providing valid data.

An inadequate hyperelastic model shows considerable variations in the response of the numerical model, as shown in the following figure in a comparison between the evaluated Yeoh models.

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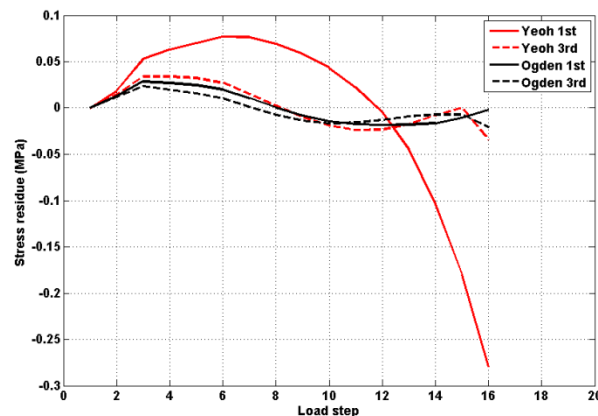


Figure 6. Comparison of waste between Yeoh and Ogden models.


4. CONCLUSIONS

In this work a numerical-experimental study of a polyurethane adhesive was performed. The main conclusions are the following:

- The results of this work show that there is more than one hyperelastic model capable of representing the behavior of a polyurethane specimen, subjected to a uniaxial stress test. Since the objective of these studies is to keep the polyurethane adhesive below the critical load, it was concluded that the 2-parameter Mooney-Rivlin model is the most suitable for numerical studies with this particular material.
- By selecting specimens whose behavior is similar, which have no anomalies, it was possible to obtain an equation describing the stress-strain graph for the material, which was used to evaluate the numerical hyperelastic models.
- According to figure 6, some models that satisfactorily describe the material, have a deviation of rigidity against experimental data, depending on the model.
- The 1st order Yeoh model shows a significant increase, discarding as a valid model. The 3rd order Yeoh model presents the most significant increase in stiffness throughout the test, among the models evaluated, with deviations greater than 4%, so its use is not recommended. The 1st and 3rd order Ogden models have a more accurate correlation; however, the 1st order Ogden model demonstrates a rigidity closer to the experiment when the load approaches the final load with a deviation of 0.66% versus 3.59% of the 3rd order model. Finally, the 2-parameter Mooney-Rivlin model exhibits an adequate behavior throughout the test, with a significant deviation close to or less than 2.5%, although it presents an increase in stiffness when approaching the ultimate load of 3.91%.
- Since the goal of these studies is to keep the polyurethane adhesive below the critical load, it was concluded that the Mooney-Rivlin 2-parameter model is best suited to perform numerical studies with this particular material.
- Future works must include shear tests to complement the mathematical models.

REFERENCES

- [1] C. Cruz, A. Yamilloux, P. Gonzalez-G, J. Taha-T, H. Gamez y R. Llerenas, «FINITE ELEMENT EVALUATION FOR ADHESIVE JOINTS DISSIMILAR MATERIAL (STEEL – ALUMINUM),» *DYNA*, vol. 93, pp. 404-410, 2018. DOI: <http://dx.doi.org/10.6036/8619>
- [2] C. E. Cruz-G, A. Akhavan-Safar, L. F. M. da Silva y M. R. Ayatollahi, «On the evaluation of a critical distance approach for failure load prediction of adhesively bonded dissimilar materials,» *Continuum Mechanics and Thermodynamics* (2020), 2020.
- [3] H. Dirajani y R. Naghdabadi, «Hyperelastic materials behavior modeling using consistent strain energy density functions,» *Acta Mechanica*, pp. 235-254, 2010.
- [4] G. Saccomandi y L.-. Vergori, «Generalised Mooney-Rivlin models for brain tissue: A theoretical perspective,» *International Journal of NonLinear Mechanics*, vol. 109, pp. 9-14, 2019.

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- [5] Abubakar, P. Myler y E. Zhou, «Constitutive Modelling of Elastomeric Seal Material under Compressive Loadin".,» *Modeling and Numerical Simulation of Material Science*, vol. 6, pp. 28-40, 2016.
- [6] R. Mangan, M. Destrade y G. Saccomandi, «Strain energy function for isotropic non-linear elastic incompressible solids with linear finite strain respons and torsion,» *Extreme Mechanics Letters*, vol. 9, pp. 204-206, 2016.
- [7] M. A. Hassan, A. Abouel-Kasem, and M. A. El-Sharief, J. "Evaluation of the material constants of nitrile butadeine rubbers (nbr) with different carbon black (cb): fe-simulation and experimental" *Eng. Sci. Assiut Univ.*, 38, 119 (2010).
- [8] Mooney-Rivlin Model, and Ogden Model for Chloroprene Rubber", *International Journal of Precision Engineering and Manufacturing*, 13, pp. 759-764. 2012.
- [9] Jaramillo H. "Evaluation of the Use of the Yeoh and Mooney-Rivlin Functions as Strain Energy Density Functions for the Ground Substance Material of the Annulus Fibrosus", *Mathematical Problems in Engineering*, 29, pp. 1-10. 2018.

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